MATH 221 · FALL 2026

Problem Set 2: Row Reduction

DUE Friday, September 11th by 11:59pm

It's time you realized that you have something in you more powerful and miraculous than the things that affect you and make you dance like a puppet.

From *Meditations* by Marcus Aurelius

Solutions must be typeset in LaTeX and written in complete, grammatical sentences. You may consult the course notes, the supplementary textbook, your classmates, your professor, math interns, or Q-Center tutors for help with the assignment, but **all sources and collaborators must be explicitly referenced**. Everyone is expected to write their own solutions. Feel free to DM on Discord if you have any questions.

PROBLEM 2.1

Use row reduction to find the general solution to each of the following systems of equations. You may use the Row Reducer to help you do the calculations. The **Export LaTeX** button shows the code for your row reduction, which you may copy and paste into your solutions.

1. $$\begin{aligned} 2y + 4z &= 10 \\ x - y + z &= 6 \\ 3x + y - 2z &= -1 \end{aligned}$$

2. $$\begin{aligned} x_3 + x_4 &= 3 \\ x_1 + 2x_2 + x_3 &= 2 \\ 3x_1 + 6x_2 + 2x_3 + 2x_4 &= 15 \end{aligned}$$

3. $$\begin{aligned} x - y + 2z &= 4 \\ 2x - 2y + 3z &= 5 \\ -x + 4y + z &= 2 \\ x + y + z &= 1 \end{aligned}$$

4. $$\begin{aligned} x_3 + x_4 + 2x_5 &= 4 \\ x_1 + 3x_2 + x_3 + x_5 &= 1 \\ 2x_1 + 6x_2 - x_3 + x_4 - 4x_5 &= 10 \\ x_1 + 3x_2 + 2x_3 + 5x_4 + 3x_5 &= 25 \end{aligned}$$

PROBLEM 2.2

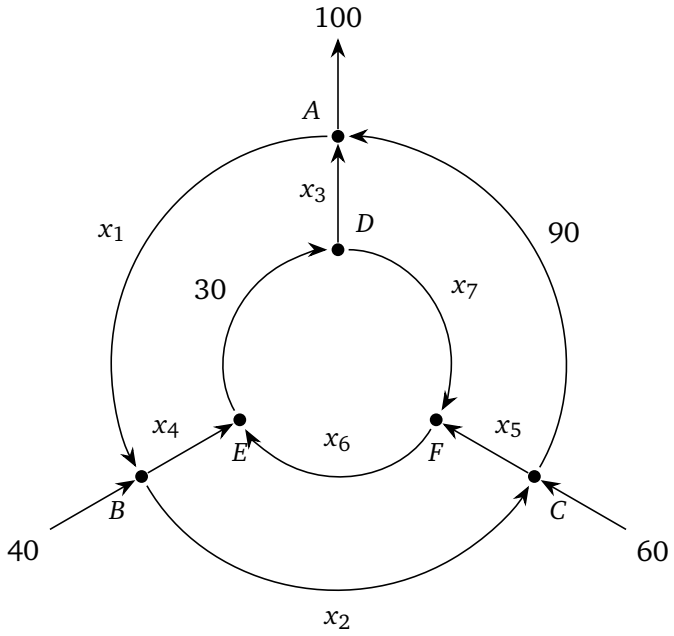
A **parabola** is a curve described by an equation of the form $y = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$ are real numbers and $a \neq 0$.

1. Find all the parabolas which pass through the points $(-1, 6)$, $(1, 4)$, $(2, 12)$ or explain why none exist.

2. Find all the parabolas which pass through the points $(-3, 2)$, $(3, 6)$, $(6, 8)$ or explain why none exist.

PROBLEM 2.3

The traffic department has turned its attention to a neighborhood with a unique traffic pattern described by the diagram below.



Based on the lessons they learned from the previous job, they added sensors to each of the inner and outer loops with the aim of getting a complete picture of the traffic flow through the neighborhood. The readings for a typical afternoon in cars per hour are labeled on the diagram.

1. The crew is having trouble solving for the flows on each street. Help them work out the general solution to the traffic flows given these readings.

2. What's the fewest number of sensors they need to add to get a complete reading of the traffic in this neighborhood? Explain.

3. Give an example of streets where the sensors can be placed to get a complete reading and an example of streets where placing the same number of sensors does not give a complete reading.