MATH 221 · FALL 2026

Problem Set 2: Row Reduction

SOLUTIONS

It's time you realized that you have something in you more powerful and miraculous than the things that affect you and make you dance like a puppet.

From *Meditations* by Marcus Aurelius

Solutions must be typeset in LaTeX and written in complete, grammatical sentences. You may consult the course notes, the supplementary textbook, your classmates, your professor, math interns, or Q-Center tutors for help with the assignment, but **all sources and collaborators must be explicitly referenced**. Everyone is expected to write their own solutions. Feel free to DM on Discord if you have any questions.

PROBLEM 2.1

Use row reduction to find the general solution to each of the following systems of equations. You may use the Row Reducer to help you do the calculations. The **Export LaTeX** button shows the code for your row reduction, which you may copy and paste into your solutions.

1. $$\begin{aligned} 2y + 4z &= 10 \\ x - y + z &= 6 \\ 3x + y - 2z &= -1 \end{aligned}$$

SOLUTION

We convert this system to an augmented matrix, and then row reduce.

SOLUTION (CONTINUED)

$$\left[\begin{array}{ccc|c} 0 & 2 & 4 & 10 \\ 1 & -1 & 1 & 6 \\ 3 & 1 & -2 & -1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & 4 & 10 \\ 3 & 1 & -2 & -1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & 4 & 10 \\ 0 & 4 & -5 & -19 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 4 & -5 & -19 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -13 & -39 \end{array}\right] \xrightarrow{R_3 \to -\frac{1}{13}R_3} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - 3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Converting back to equations we have a unique solution $(x, y, z) = (2, -1, 3)$. □

2. $$\begin{aligned} x_3 + x_4 &= 3 \\ x_1 + 2x_2 + x_3 &= 2 \\ 3x_1 + 6x_2 + 2x_3 + 2x_4 &= 15 \end{aligned}$$

SOLUTION

We convert this system to an augmented matrix, and then row reduce.

SOLUTION (CONTINUED)

$$\left[\begin{array}{cccc|c} 0 & 0 & 1 & 1 & 3 \\ 1 & 2 & 1 & 0 & 2 \\ 3 & 6 & 2 & 2 & 15 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 3 & 6 & 2 & 2 & 15 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & 2 & 9 \end{array}\right] \xrightarrow{R_1 \to R_1 + R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 11 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & 2 & 9 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 + R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 11 \\ 0 & 0 & 0 & 3 & 12 \\ 0 & 0 & -1 & 2 & 9 \end{array}\right] \xrightarrow{R_3 \to -R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 11 \\ 0 & 0 & 0 & 3 & 12 \\ 0 & 0 & 1 & -2 & -9 \end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 11 \\ 0 & 0 & 1 & -2 & -9 \\ 0 & 0 & 0 & 3 & 12 \end{array}\right] \xrightarrow{R_3 \to \frac{1}{3} R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 11 \\ 0 & 0 & 1 & -2 & -9 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - 2R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & -2 & -9 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \to R_2 + 2R_3} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

The lack of a pivot in the second column tells us that x_2 is a free variable. Converting back to equations we have

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ x_3 &= -1 \\ x_4 &= 4 \end{aligned}$$

Solving for the dependent variables and replacing x_2 with a parameter gives us the general solution

$$(x_1, x_2, x_3, x_4) = (3 - 2t, t, -1, 4).$$ □

3. $$\begin{aligned} x - y + 2z &= 4 \\ 2x - 2y + 3z &= 5 \\ -x + 4y + z &= 2 \\ x + y + z &= 1 \end{aligned}$$

SOLUTION

We convert this system to an augmented matrix, and then row reduce.

SOLUTION (CONTINUED)

$$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 2 & -2 & 3 & 5 \\ -1 & 4 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 0 & 0 & -1 & -3 \\ -1 & 4 & 1 & 2 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 0 & 0 & -1 & -3 \\ 0 & 3 & 3 & 6 \\ 1 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_4 \to R_4 - R_1} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 0 & 0 & -1 & -3 \\ 0 & 3 & 3 & 6 \\ 0 & 2 & -1 & -3 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 0 & 3 & 3 & 6 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & -1 & -3 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{3} R_2} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & -1 & -3 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 2 & -1 & -3 \end{array}\right] \xrightarrow{R_4 \to R_4 - 2R_2} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -7 \end{array}\right]$$

$$\xrightarrow{R_3 \to -R_3} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -7 \end{array}\right] \xrightarrow{R_1 \to R_1 - 3R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -7 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -3 & -7 \end{array}\right] \xrightarrow{R_4 \to R_4 + 3R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

The final row of this matrix converts to the equation $0 = 2$, which is a contradiction. Thus this system is **inconsistent**. □

4.
$$\begin{aligned} x_3 + x_4 + 2x_5 &= 4 \\ x_1 + 3x_2 + x_3 + x_5 &= 1 \\ 2x_1 + 6x_2 - x_3 + x_4 - 4x_5 &= 10 \\ x_1 + 3x_2 + 2x_3 + 5x_4 + 3x_5 &= 25 \end{aligned}$$

SOLUTION

We convert this system to an augmented matrix, and then row reduce.

SOLUTION (CONTINUED)

$$\left[\begin{array}{ccccc|c} 0 & 0 & 1 & 1 & 2 & 4 \\ 1 & 3 & 1 & 0 & 1 & 1 \\ 2 & 6 & -1 & 1 & -4 & 10 \\ 1 & 3 & 2 & 5 & 3 & 25 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccccc|c} 1 & 3 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 2 & 6 & -1 & 1 & -4 & 10 \\ 1 & 3 & 2 & 5 & 3 & 25 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccccc|c} 1 & 3 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & -3 & 1 & -6 & 8 \\ 1 & 3 & 2 & 5 & 3 & 25 \end{array}\right] \xrightarrow{R_4 \to R_4 - R_1} \left[\begin{array}{ccccc|c} 1 & 3 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & -3 & 1 & -6 & 8 \\ 0 & 0 & 1 & 5 & 2 & 24 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -1 & -1 & -3 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & -3 & 1 & -6 & 8 \\ 0 & 0 & 1 & 5 & 2 & 24 \end{array}\right] \xrightarrow{R_3 \to R_3 + 3R_2} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -1 & -1 & -3 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 4 & 0 & 20 \\ 0 & 0 & 1 & 5 & 2 & 24 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 - R_2} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -1 & -1 & -3 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 4 & 0 & 20 \\ 0 & 0 & 0 & 4 & 0 & 20 \end{array}\right] \xrightarrow{R_3 \to \frac{1}{4} R_3} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & -1 & -1 & -3 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 4 & 0 & 20 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 4 & 0 & 20 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_3} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 4 & 0 & 20 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 - 4R_3} \left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Columns 2 and 5 lack pivots, hence x_2 and x_5 are free variables. Converting back to equations we have

$$\begin{aligned} x_1 + 3x_2 - x_5 &= 2 \\ x_3 + 2x_5 &= -1 \\ x_4 &= 5 \end{aligned}$$

Solving for the dependent variables and replacing x_2 and x_5 by parameters gives us the general solution

$$(x_1, x_2, x_3, x_4, x_5) = (-3s + t + 2, x, -2t - 1, 5, t). \qquad \square$$

PROBLEM 2.2

A **parabola** is a curve described by an equation of the form $y = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$ are real numbers and $a \neq 0$.

1. Find all the parabolas which pass through the points $(-1, 6)$, $(1, 4)$, $(2, 12)$ or explain why none exist.

SOLUTION

Each point gives us a linear equation satisfied by the coefficients a, b, c. These equations are

$$\begin{aligned} a - b + c &= 6 \\ a + b + c &= 4 \\ 4a + 2b + c &= 12. \end{aligned}$$

Converting to an augmented matrix and row reducing gives us

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 1 & 4 \\ 4 & 2 & 1 & 12 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & 0 & -2 \\ 4 & 2 & 1 & 12 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_1} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 2 & 0 & -2 \\ 0 & 6 & -3 & -12 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & 0 & -1 \\ 0 & 6 & -3 & -12 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 6 & -3 & -12 \end{array}\right] \xrightarrow{R_3 \to R_3 - 6R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -3 & -6 \end{array}\right]$$

$$\xrightarrow{R_3 \to -\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Thus there is a unique parabola passing through these points, namely

$$y = 3x^2 - x + 2.$$ □

2. Find all the parabolas which pass through the points $(-3, 2)$, $(3, 6)$, $(6, 8)$ or explain why none exist.

SOLUTION

SOLUTION (CONTINUED)

We setup a similar system of equations as in the previous part

$$\begin{aligned} 9a - 3b + c &= 2 \\ 9a + 3b + c &= 6 \\ 36a + 6b + c &= 8. \end{aligned}$$

Row reduction gives

$$\left[\begin{array}{ccc|c} 9 & -3 & 1 & 2 \\ 9 & 3 & 1 & 6 \\ 36 & 6 & 1 & 8 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 9 & -3 & 1 & 2 \\ 0 & 6 & 0 & 4 \\ 36 & 6 & 1 & 8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_1} \left[\begin{array}{ccc|c} 9 & -3 & 1 & 2 \\ 0 & 6 & 0 & 4 \\ 0 & 18 & -3 & 0 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{9} R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{9} & \frac{2}{9} \\ 0 & 6 & 0 & 4 \\ 0 & 18 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{9} & \frac{2}{9} \\ 0 & 6 & 0 & 4 \\ 0 & 0 & -3 & -12 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{6} R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{9} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & -3 & -12 \end{array}\right]$$

$$\xrightarrow{R_3 \to -\frac{1}{3} R_3} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{1}{9} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1 \to R_1 + \frac{1}{3} R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{9} & \frac{4}{9} \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{9} R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 4 \end{array}\right]$$

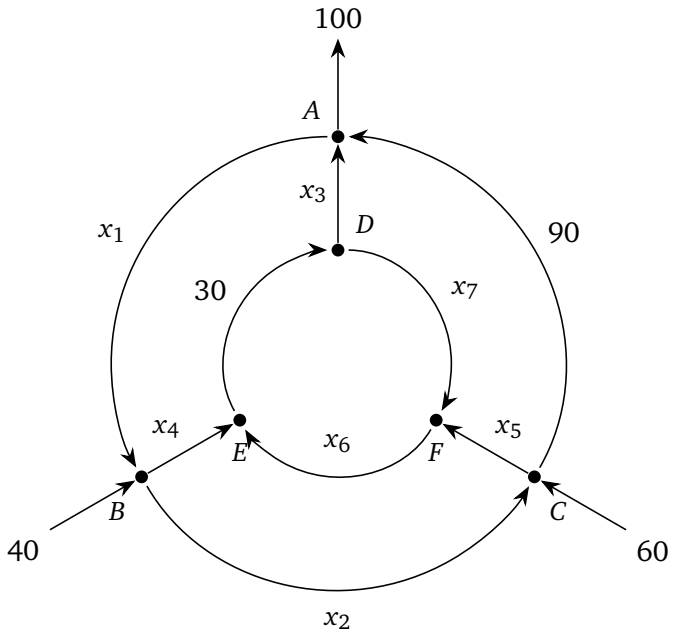
Thus there is a unique solution which corresponds to the curve

$$y = \frac{2}{3}x + 4.$$

Since $a = 0$, this is a line, not a parabola. Hence there are no parabolas passing through these three points. □

PROBLEM 2.3

The traffic department has turned its attention to a neighborhood with a unique traffic pattern described by the diagram below.



Based on the lessons they learned from the previous job, they added sensors to each of the inner and outer loops with the aim of getting a complete picture of the traffic flow through the neighborhood. The readings for a typical afternoon in cars per hour are labeled on the diagram.

1. The crew is having trouble solving for the flows on each street. Help them work out the general solution to the traffic flows given these readings.

SOLUTION

Using the observation that the total traffic into and out of an intersection must be the same, we get the following system of equations

$$\begin{aligned}
A: \quad x_3 + 90 &= x_1 + 100 \\
B: \quad x_1 + 40 &= x_2 + x_4 \\
C: \quad x_2 + 60 &= x_5 + 90 \\
D: \quad 30 &= x_3 + x_7 \\
E: \quad x_4 + x_6 &= 30 \\
F: \quad x_5 + x_7 &= x_6
\end{aligned}$$

Moving all the constants to one side of the equation and all the variables to the other, we get

$$\begin{aligned}
A: \quad x_1 - x_3 &= -10 \\
B: \quad -x_1 + x_2 + x_4 &= 40 \\
C: \quad -x_2 + x_5 &= -30 \\
D: \quad x_3 + x_7 &= 30 \\
E: \quad x_4 + x_6 &= 30 \\
F: \quad -x_5 + x_6 - x_7 &= 0
\end{aligned}$$

SOLUTION (CONTINUED)

This system of equations corresponds to the following augmented matrix

$$\left[\begin{array}{rrrrrrr|r} 1 & 0 & -1 & 0 & 0 & 0 & 0 & -10 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 40 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & -30 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 \end{array}\right].$$

Row reducing we end up with

$$\left[\begin{array}{rrrrrrr|r} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 20 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 30 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 30 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Since this matrix is in RREF and there are twp non-augmented columns without a pivot, there are two free variables. Converting back into equations we get the general solution,

$$\begin{aligned} x_1 &= 20 - t \\ x_2 &= 30 + s - t \\ x_3 &= 30 - t \\ x_4 &= 30 - s \\ x_5 &= s - t \\ x_6 &= s \\ x_7 &= t. \end{aligned}$$

Note that choosing any s and t with $0 < t < s < 30$ and $t < 20$ gives a solution in which every street carries a positive flow. For example, $(s, t) = (20, 10)$ gives the flows $(x_1, \ldots, x_7) = (10, 40, 20, 10, 10, 20, 10)$. □

2. What's the fewest number of sensors they need to add to get a complete reading of the traffic in this neighborhood? Explain.

SOLUTION

Two sensors are needed in order to measure the traffic flow, one for each free variable. Any fewer than two measurements will not give us enough data to solve uniquely for s and t. □

3. Give an example of streets where the sensors can be placed to get a complete reading and an example of streets where placing the same number of sensors does not give a complete reading.

SOLUTION

We can place the sensors on streets 6 and 7 in order to directly measure s and t, then all the rest of the traffic flows are determined. However, sensors on, say, streets 4 and 6 will not give a complete reading because those traffic flows are completely determined by s. Similarly, streets 2 and 5 are not a good pair because they only allow us to determine $s - t$, which is not enough info to determine s and t separately. □