MATH 221 · FALL 2026

Problem Set 5: Subspaces, Bases, and Dimension

DUE Friday, October 9th by 11:59pm

I call our world Flatland, not because we call it so, but to make its nature clearer to you, my happy readers, who are privileged to live in Space.

From *Flatland* by Edwin A. Abbott

Solutions must be typeset in LaTeX and written in complete, grammatical sentences. You may consult the course notes, the supplementary textbook, your classmates, your professor, math interns, or Q-Center tutors for help with the assignment, but **all sources and collaborators must be explicitly referenced**. Everyone is expected to write their own solutions. Feel free to DM on Discord if you have any questions. Problems marked (Challenge) are optional and are for extra credit

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PROBLEM 5.1

Let

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 1 & 1 & 1 & 2 \\ 2 & -1 & 5 & -5 \end{bmatrix}.$$

You may use the Row Reducer to help you calculate.

1. Find a basis for $\text{Nul}(A)$. What is $\dim \text{Nul}(A)$?
2. Find a basis for $\text{Col}(A)$. What is $\dim \text{Col}(A)$?
3. Is the vector $b = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$ in $\text{Col}(A)$?
4. Is the vector $v = \begin{bmatrix} -3 \\ 4 \\ 1 \\ -1 \end{bmatrix}$ in $\text{Nul}(A)$?
5. (Challenge) Find an equation in b_1, b_2, b_3 that is satisfied exactly when the vector $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ belongs to the column space $\text{Col}(A)$.

PROBLEM 5.2

For each of the following collections of vectors, decide whether it is a basis of $\mathbb{R}^3$. Explain your answer.

1. $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}$
2. $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\0\\1\end{bmatrix}$, $v_4 = \begin{bmatrix}1\\1\\1\end{bmatrix}$
3. $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}$
4. $v_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\2\\1\end{bmatrix}$

PROBLEM 5.3

The vectors

$$v_1 = \begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \qquad v_2 = \begin{bmatrix}0\\1\\1\\0\end{bmatrix}$$

are linearly independent.

1. Explain why v_1, v_2 is not a basis of $\mathbb{R}^4$.
2. Find a basis of $\mathbb{R}^4$ that includes v_1 and v_2.

PROBLEM 5.4

Let $V = \text{Span}\{v_1, v_2\}$, where

$$v_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}, \qquad v_2 = \begin{bmatrix}1\\-1\\2\end{bmatrix},$$

and let $\mathcal{B} = \{v_1, v_2\}$.

1. Explain why $\mathcal{B}$ is a basis of V.
2. The vector $w = \begin{bmatrix}2\\4\\-2\end{bmatrix}$ is in V. Find $[w]_{\mathcal{B}}$.
3. Is the vector $u = \begin{bmatrix}1\\2\\3\end{bmatrix}$ in V? If it is, find $[u]_{\mathcal{B}}$.