MATH 301 · FALL 2026

Problem Set 1: Warm Up

DUE Friday, September 4th at 11:59pm

The journey of a thousand miles begins with one step.

From *Dao De Jing* by Laozi

Solutions must be typeset in LaTeX and written in complete, grammatical sentences. **You must list all your collaborators explicitly**. Feel free to DM on Discord if you have any questions. Problems marked (Challenge) are optional but encouraged.

PROBLEM 1.1

In this problem we review doing calculations in symmetric groups. **You do not need to show any work, just the final answers.**

1. Below are compositions of permutations expressed in cycle notation. Express each permutation as a collection of disjoint cycles; that is, calculate the cycle notation for the composite permutation.
   (a) $(134)(241)(1234)$
   (b) $(15)(14)(13)(12)$
   (c) $(15342)(43512)$
2. Calculate the order of the following permutations. Beware, I may be trying to trick you!
   (a) $(123)(45)(67)$
   (b) $(325)(146)(27)$
3. For each of the following permutations σ, determine the inverse permutation σ^{-1}.
   (a) (12345)
   (b) $(123)(456)$
   (c) $(12)(345)(6789)$

   Can you see a general recipe for finding the inverse of a permutation expressed in cycle notation?

PROBLEM 1.2

1. Give an example of an abelian group that is not cyclic (and prove it).
2. Find two non-isomorphic groups of order 6 and prove they are not isomorphic.

3. What are the possible orders of elements in the symmetric group S_5? For each possible order, find an element in S_5 with that order.

4. If $H \subseteq G$ is a subgroup with index 2, prove that H is a normal subgroup of G.

5. Let D_6 be the 6th dihedral group (with order 12). Determine all the conjugacy classes of elements of D_6.

PROBLEM 1.3

The table below gives the partial description of a mystery group G of order 8.

G	1	a	b	c	d	e	f	g
1								
a		1				d		
b			a		f			
c		b						
d			g					
e		d						
f								
g								a

1. Using the axioms of a group and information provided, fill in the missing entries of the multiplication table for G. There is only one solution!

2. Is G isomorphic to D_4? Either find an isomorphism or prove that no such isomorphism exists.

PROBLEM 1.4

Let G be a group and suppose that $H_1, H_2 \subseteq G$ are subgroups such that $H_i \neq G$.

1. Prove that $G \neq H_1 \cup H_2$.

2. Find an example of a group G which can be expressed as a union of three proper subgroups $G = H_1 \cup H_2 \cup H_3$.

3. (Challenge) Let G be a group. Prove that there are three proper subgroups $H_1, H_2, H_3 \subseteq G$ such that $G = H_1 \cup H_2 \cup H_3$ if and only if there is a surjective homomorphism $\rho : G \to \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

PROBLEM 1.5 · (Challenge)

Let p be an odd prime and suppose that G is a finite group of order $2p$. Prove that G is either cyclic or $G \cong D_p$, the pth dihedral group.